§6. Toroidal Viscosity in Plateau Collisionality Regime in Toroidal Systems

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In order to evaluate the bootstrap current, the parallel force balance is solved, where parallel viscosities and parallel frictions are balanced. In contrast with it, toroidal and/or poloidal viscosities are useful in the evaluation of the plasma rotation.

In the plateau collisionality regime, for arbitrary divergence free vector $\vec{A}$ ($\nabla \cdot \vec{A} = 0$) the following relations are obtained in the Boozer coordinate system (ψ, θ, ζ):

$$\begin{bmatrix} \left\langle \vec{A} \cdot \nabla \cdot \overleftrightarrow{\Pi}_a \right\rangle \\ -\left\langle \vec{A} \cdot \nabla \cdot \overleftrightarrow{\Theta}_a \right\rangle \end{bmatrix} = \begin{bmatrix} \mu_{a1} & \mu_{a2} \\ \mu_{a2} & \mu_{a3} \end{bmatrix} \begin{bmatrix} \langle \vec{u}_a \cdot \nabla\theta^* \rangle \\ -\dfrac{2}{5P_a} \langle \vec{q}_a \cdot \nabla\theta^* \rangle \end{bmatrix}$$

where

$$\theta^* = (I + \langle G \rangle)\theta + (J - \iota \langle G \rangle)\zeta,$$

$$\mu_{aj} = \frac{n_a m_a}{\tau_{aa}} \alpha_j \frac{\lambda_a}{\lambda_{PL}},$$

$$\alpha_1 = 2, \quad \alpha_2 = -1, \quad \alpha_3 = \frac{13}{2},$$

$$\frac{1}{\lambda_{PL}} = \frac{\sqrt{\pi} \left\langle \dfrac{\vec{A} \cdot \nabla B}{B} \displaystyle\sum_{mn} \frac{C_{mn} \exp^{i(m\theta + n\zeta)}}{|n + \iota m|} \right\rangle}{2 \left\langle \dfrac{1}{\sqrt{g}} \right\rangle},$$

$$\langle G \rangle = \frac{\left\langle \dfrac{\vec{A} \cdot \nabla B}{B} \displaystyle\sum_{mn} \frac{D_{mn} \exp^{i(m\theta + n\zeta)}}{|n + \iota m|} \right\rangle}{\left\langle \dfrac{\vec{A} \cdot \nabla B}{B} \displaystyle\sum_{mn} \frac{C_{mn} \exp^{i(m\theta + n\zeta)}}{|n + \iota m|} \right\rangle},$$

$$C_{mn} = \left[\frac{\hat{n} \cdot \nabla B}{B} \right]_{mn},$$

$$D_{mn} = \left[\frac{\hat{n} \cdot \nabla \left(B^2 \langle g_2 \rangle - \langle B^2 \rangle g_2 \right)}{2B^2} \right]_{mn},$$

$$[F]_{mn} \equiv \frac{1}{(2\pi)^2} \int_0^{2\pi} d\theta \int_0^{2\pi} d\zeta F \exp^{-i(m\theta + n\zeta)},$$

$$\sqrt{g} = \frac{J + \iota I}{B^2}$$

Here, $2\pi J$ is the poloidal current outside of a flux surface, $2\pi I$ is the toroidal current inside, and the function g_2 is the solution of the following magnetic differential equation:

$$\vec{B} \cdot \nabla \left(\frac{g_2}{B^2} \right) = \vec{B} \times \nabla\psi \cdot \nabla \left(\frac{1}{B^2} \right),$$

$$g_2(B = B_{max}) = 0.$$

In the Boozer coordinate system, the magnetic field is expressed by

$$\vec{B} = \vec{B}_T + \vec{B}_P,$$

$$\vec{B}_T = \nabla\psi \times \nabla\theta, \quad \vec{B}_P = \iota \nabla\zeta \times \nabla\psi.$$

Thus, we can obtain the parallel, toroidal, and poloidal viscosities, setting $\vec{A} = \vec{B}$, $\vec{A} = \vec{B}_T$, and $\vec{A} = \vec{B}_P$, respectively. It follows that

$$\begin{bmatrix} \left\langle \vec{B} \cdot \nabla \cdot \overleftrightarrow{\Pi}_a \right\rangle \\ -\left\langle \vec{B} \cdot \nabla \cdot \overleftrightarrow{\Theta}_a \right\rangle \end{bmatrix} = \begin{bmatrix} \left\langle \vec{B}_T \cdot \nabla \cdot \overleftrightarrow{\Pi}_a \right\rangle \\ -\left\langle \vec{B}_T \cdot \nabla \cdot \overleftrightarrow{\Theta}_a \right\rangle \end{bmatrix} + \begin{bmatrix} \left\langle \vec{B}_P \cdot \nabla \cdot \overleftrightarrow{\Pi}_a \right\rangle \\ -\left\langle \vec{B}_P \cdot \nabla \cdot \overleftrightarrow{\Theta}_a \right\rangle \end{bmatrix}$$

Also, it is clear that when there is axisymmetry, i.e., $\partial/\partial\zeta = 0$

$$\begin{bmatrix} \left\langle \vec{B}_T \cdot \nabla \cdot \overleftrightarrow{\Pi}_a \right\rangle \\ -\left\langle \vec{B}_T \cdot \nabla \cdot \overleftrightarrow{\Theta}_a \right\rangle \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

The toroidal viscosity is used in the comparison of the neoclassical theory with the experimental results in Compact Helical System (CHS). The toroidal viscosity clearly reflects the change of the toroidal bummpyness due to the Fourier mode with $(m, n) = (0, 8)$ according to the vacuum magnetic axis shift.

The extention to other collisionality regimes will be performed. Moreover, the extension to the case where large poloidal flow exists will be performed. By such extensions, we can examine the possibility of H-mode transition more presicely.